Menu

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Enrichment

8.7 Conjugate Radicals

You have seen the term *conjugate* applied to complex numbers of the form $a + bi$ and $a - bi$. This term can also be applied to other pairs of expressions containing two terms that differ only in the sign of the second term. In the exercises that follow, you will explore conjugate radical expressions.

In Exercises 1–4, write the conjugate of the given radical expression.

1. $3 + 5\sqrt{2}$ ______________
2. $-3 - 5\sqrt{2}$ ______________
3. $-5 + 2\sqrt{3}$ ______________
4. $2\sqrt{3} - 5\sqrt{2}$ ______________

In Exercises 5–10, a and b are positive rational numbers such that $\sqrt{a}$ and $\sqrt{b}$ are irrational. Perform the indicated operation, and tell whether the result is rational or irrational.

5. $(a + \sqrt{b}) + (a - \sqrt{b})$ ______________ ______________
6. $(a + \sqrt{b}) - (a - \sqrt{b})$ ______________ ______________
7. $(a + \sqrt{b}) \cdot (a - \sqrt{b})$ ______________ ______________
8. $(\sqrt{a} + \sqrt{b}) + (\sqrt{a} - \sqrt{b})$ ______________ ______________
9. $(\sqrt{a} + \sqrt{b}) - (\sqrt{a} - \sqrt{b})$ ______________ ______________
10. $(\sqrt{a} + \sqrt{b}) \cdot (\sqrt{a} - \sqrt{b})$ ______________ ______________

To learn how to rationalize the denominator of a fraction such as $\dfrac{9}{3 - \sqrt{3}}$, examine the steps below.

$$\frac{9}{3-\sqrt{3}} = \frac{9}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}$$

$$= \frac{9(3+\sqrt{3})}{9-3}$$

$$= \frac{9(3+\sqrt{3})}{6} = \frac{3(3+\sqrt{3})}{2}$$

In Exercises 11–13, rationalize the denominator of the given fraction, and simplify as needed.

11. $\dfrac{12}{\sqrt{6} - 2}$ ______________
12. $\dfrac{6 + \sqrt{3}}{4 - \sqrt{3}}$ ______________
13. $\dfrac{2\sqrt{5} + 3\sqrt{2}}{3\sqrt{5} - \sqrt{2}}$ ______________

ANSWERS

Lesson 8.5

1. $\frac{5}{x+1} - \frac{2}{x+4}$
2. $\frac{1}{x} + \frac{2}{x+2} + \frac{3}{x-2}$
3. $\frac{5}{x+1} + \frac{3}{x-2} - \frac{1}{x+3}$
4. $\frac{-1}{x-1} - \frac{3}{(x-1)^2} + \frac{2}{x-2}$

Lesson 8.6

1. a. about 181 kilograms
   b. about 1376 kilograms
   c. $m_v = \frac{mc\sqrt{c^2 - v^2}}{c^2 - v^2}$
2. 11.3 kilometers per second
3. $d = \sqrt{2rh + h^2}$
4. about 896.1 miles
5. 84 square inches

Lesson 8.7

1. $3 - 5\sqrt{2}$
2. $-3 + 5\sqrt{2}$
3. $-5 - 2\sqrt{3}$
4. $2\sqrt{3} + 5\sqrt{2}$
5. $2a$; rational
6. $2\sqrt{b}$; irrational
7. $a^2 - b$; rational
8. $2\sqrt{a}$; irrational
9. $2\sqrt{b}$; irrational
10. $a - b$; rational
11. $6(2 + \sqrt{6})$
12. $\frac{27 + 10\sqrt{3}}{13}$
13. $\frac{36 + 11\sqrt{10}}{43}$

Lesson 8.8

1. a. $\frac{17 + \sqrt{369}}{2} \approx 18.1$ and
   $\frac{17 - \sqrt{369}}{2} \approx -1.1$
   b. -1.1 is valid and 18.1 is extraneous.
   c. Answers may vary. Sample answer: 18.1 seems close to the value of x for which $g(x) = 4$.
2. a. Same possible solutions as for $f(x) = 4$, -1.1 and 18.1.
   b. 18.1 is valid and -1.1 is extraneous.
3. a. Both produce the same quadratic equation, $x^2 + x - 11 = 0$.
   b. $\frac{-1 + \sqrt{45}}{2} \approx 2.9$ or $\frac{-1 - \sqrt{45}}{2} \approx -3.9$.
   Both solutions are extraneous for $f(x) = -5$; both solutions are valid for $g(x) = -5$.
4. a. Both produce $x^2 + 7x + 28 = 0$.
   b. The roots are imaginary. Justifications may vary. Sample answer: The discriminant is negative.
   c. Answers may vary. Sample answer: The line $y = -8$ does not intersect either graph.

Enrichment — Chapter 9

Lesson 9.1

1. $x = 6$
2. $y = -1$
3. $7x - 5y = 4$
4. a line that is parallel to ℓ and m and midway between them
5. $y = 4$
6. $x = -2$
7. $y = x + 4$
8. a circle with its center at C and a radius of 4
9. a circle with its center at C and a radius of r